

Boltzmann equation for fluctuation Cooper pairs in Lawrence-Doniach model. Possible out-of-plane negative differential conductivity

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 (Dated: February 5, 2008)

The differential conductivity for the out-of-plane transport in layered cuprates is calculated for Lawrence-Doniach model in the framework of time-dependent Ginzburg-Landau (TDGL) theory. The TDGL equation for the superconducting order parameter is solved in the presence of Langevin external noise, describing the birth of fluctuation Cooper pairs. The TDGL correlator of the superconducting order parameter is calculated in momentum representation and it is shown that the so defined number of particles obeys the Boltzmann equation. The fluctuation conductivity is given by an integral over the Josephson phase θ of the particles distribution, depending on that phase, $n(\theta)$ and their velocity $v(\theta)$. It is demonstrated that in case of overcooling under T_c the transition from normal to superconducting phase while reducing the external electric field runs via annulation of the differential conductivity. The presented results can be used for analysis and experimental data processing of measurements of differential conductivity and fluctuation current in strong electric and magnetic fields. The possible usage of a negative differential conductivity (NDC) for generation of THz oscillations is shortly discussed.

PACS numbers: 74.40.+k, 74.20.De, 74.25.Fy, 74.72.-h

I. INTRODUCTION

Fluctuation phenomena in high-temperature cuprates (CuO_2) are greatly pronounced because of the short length of coherence. The number of degrees of freedom in the sample's volume is $\mathcal{V}/\xi_{ab}^2(0)\xi_c(0)$. The fluctuations in strongly anisotropic superconductors are approximately two-dimensional in layers at a distance s from each other and the number of fluctuation modes is $\mathcal{V}/\xi_{ab}^2(0)s$. A reduction of the dimension, as a rule, leads to enhancement of the fluctuation effects. The paraconductivity of Aslamazov-Larkin (AL), when to the normal conductivity we have to add the contribution of the fluctuation Cooper pairs is the most thoroughly investigated fluctuation phenomenon. The AL conductivity is easier to be observed if the normal phase conductivity is small. That is the case of a strongly disordered conventional superconductor or high-temperature cuprates, containing CuO_2 planes as a main structural detail. The amplitudes of the electron hopping between CuO_2 layers are small, which leads to intensive diminution of the conductivity in the so called dielectric z -direction, perpendicular to the CuO_2 layers. As a consequence one could apply considerable electric fields that will not result in significant heating of the sample. The small heating gives the opportunity to investigate how the strong electric field's nonlinear effects influence the fluctuation conductivity σ . Such investigations could reveal new details in the kinetics of the superconducting order parameter. They could specify the parameters of time-dependent Ginzburg-Landau (TDGL) theory which describes, with acceptable accuracy, the fluctuation phenomena in almost all superconductors. We recommend the monograph by Larkin and Varlamov¹ for a general review on fluctuation phenomena

in superconductors and the review article (and references therein)² which is especially devoted to Gaussian fluctuations in layered superconductors. In addition, the recent experimental investigations on cut-off effects³ and reduction of paraconductivity with increase of the electric field in cuprates⁴ are also recommended.

The purpose of the current work is to demonstrate that negative differential conductivity (NDC) can be reached in the specific nonequilibrium conditions of a supercooled below T_c superconductor in normal state in external electric field E_c . This NDC can find possible applications in the usage of layered cuprates as active media in THz generators. We derived explicit formulas for differential conductivity in out of plane direction and for the current in case of external magnetic field B_z . In the special case of evanescent magnetic field our results agree with the work by Puica and Lang.⁵ Our TDGL result for the momentum distribution of the fluctuation Cooper pairs obey the Boltzmann equation. The new results for the current j_z can be used for the experimental data processing and interpretation of the experimental results for out-of-plane fluctuation conductivity for samples in the geometry of plane capacitor with a layered superconductor between the plates. Special attention is paid to the analysis of formulas for the differential conductivity in different physical conditions. How to reach easily the regime of high frequency oscillations is briefly considered.

For this sake the time dependence of the order parameter is estimated under Langevin approach and its correlator in coinciding time arguments is calculated. We have demonstrated that this correlator in case of homogeneous electric field obeys the master equation. In this way Boltzmann equation for the fluctuation Cooper pairs has been derived, which solution gives the momentum dis-

tribution of the superconducting order parameter. The electric current is represented as an integral over this distribution and the velocity. The derived formula for the current could be generalized for a self-consistent treatment of the interaction between the fluctuations. Such a possibility has been discussed in short. We have shown that when $T < T_c$ the phase transition to the superconducting state passes through the critical point of zero differential conductivity $\sigma_{\text{diff}} = 0$. This qualitative result is one of the important consequences predicted by the theory which can be described by the deduced formula for the current $j_z(\epsilon, \mathcal{P}, h)$ as a function of the dimensionless electric field $\mathcal{P} = eE_z\tau_0 s/\hbar$, dimensionless magnetic field $h = 2\pi\xi_{ab}^2(0)B_z/\Phi_0$ and the reduced temperature $\epsilon \equiv \ln(T/T_c) \approx (T - T_c)/T_c$ Eq. (65). Further decrease of the electric field E in the regime $T < T_c$ could originate either electric oscillations or lead to formation of domain structure, but for sure the homogeneous phase loses stability and a new physics takes place. What we do is to offer systematical measurements of the generation of harmonics for investigation the role of strong electric fields. The theoretical analysis is completed in the framework of the TDGL theory applied to the Lawrence-Doniach (LD) model for a layered superconductor, introduced in the next section.

II. MODEL

Our starting point is the TDGL equation, which in momentum space takes the form

$$[\varepsilon(\mathbf{P} - e^*\mathbf{A}) + a_0\epsilon]\psi_p(t) = -2a_0\tau_0[d_t\psi_p(t) - \zeta_p(t)], \quad (1)$$

where the argument of the kinetic energy is the gauge invariant kinetic momentum

$$\mathbf{p}(t) = \mathbf{P} - e^*\mathbf{A}(t), \quad (2)$$

t is the time, and τ_0 is the TDGL relaxation time. The canonic one $\mathbf{P} = \text{const}$ is a conserving quantity for a free particle moving in an external electromagnetic field. In LD model the energy spectrum is given by

$$\varepsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m_{ab}} + \frac{a_0 r}{2}(1 - \cos\theta). \quad (3)$$

Here we have used standard notations for the time-dependent vector potential of the homogeneous electric field perpendicular to the layers in our analysis $E_z(t) = -d_t A_z(t)$, the charge of the Cooper pairs $|e^*| = 2|e|$, their effective mass in the plane of the layers for the uniaxial superconductors which we study m_{ab} , the effective mass for motion in c -direction m_c and the GL parameter

$$a_0 = \frac{\hbar^2}{2m_{ab}\xi_{ab}^2(0)} = \frac{\hbar^2}{2m_c\xi_c^2(0)}, \quad \frac{a_0 r}{2} = \frac{\hbar^2}{m_c s^2}. \quad (4)$$

For high values of the LD parameter $r = (2\xi_c(0)/s)^2$ their model simply describes the anisotropic GL theory,

applied to an uniaxial crystal ($m_a = m_b \ll m_c$). Conversely, when the coherence length in perpendicular direction $\xi_c(0)$ is much smaller than the distance between the layers we have a system of two-dimensional superconductors with a Josephson coupling.

The gauge invariant Josephson's phase

$$\theta(t) = \frac{p_z s}{\hbar} = \frac{P_z - e^* A_z(t)}{\hbar/s} \in (-\pi, +\pi) \quad (5)$$

actually is the kinetic momentum of the Cooper pair measured in units, connected to the lattice constant in z direction s . For the sake of simplicity from now on we are going to omit the z index, which would be understood by the context. For the relaxation time τ_0 in the TDGL equation, the Bardeen-Cooper-Schrieffer (BCS) theory gives

$$\frac{\hbar}{2\tau_0} = \frac{8T_c}{\pi}. \quad (6)$$

This result for the microscopic theory is extremely robust. The same value we have for dirty bulk superconductors with isotropic gap and clean two-dimensional d-type superconductors.¹ Experimental researches in high- T_c cuprates confirmed this value within the experimental accuracy. For a discussion of different methods for determination of the relaxation time τ_0 , and references of relevant experimental works see Sec. 4.2 and Sec. 4.3 of the review Ref. [2]. That is why TDGL theory does not need modifications when applied for d-wave layered cuprates.

The GL theory is applicable for small by absolute value reduced temperature $|\epsilon| \ll 1$, whereas we are going to use the negative values of this parameter for a description of an overcooled superconductor plugged in an external electric field. For the Langevin term in the TDGL theory we have a white noise correlator

$$\langle \zeta_{p_1}^*(t_1) \zeta_{p_2}(t_2) \rangle = \frac{T}{a_0 \tau_0} \delta_{p_1, p_2} \delta(t_1 - t_2). \quad (7)$$

Analyzing bulk properties of the material we formally assume periodic boundary conditions for a great distance $L \gg s$, so that the quasi-momentum takes the discrete form

$$p_z = \frac{2\pi\hbar}{L}(\text{integer}) \in \left(-\frac{\pi\hbar}{s}, +\frac{\pi\hbar}{s}\right). \quad (8)$$

Let us introduce convenient for this task dimensionless variables: $u = t/\tau$ for the dimensionless time and

$$\omega(\mathbf{k}, \theta) = \frac{\varepsilon(\mathbf{p})}{a_0} = k_x^2 + k_y^2 + \frac{r}{2}(1 - \cos\theta) \quad (9)$$

for the dimensionless kinetic energy, where

$$k_x = \frac{p_x \xi_a(0)}{\hbar}, \quad k_y = \frac{p_y \xi_b(0)}{\hbar}, \quad k_z = \frac{p_z \xi_c(0)}{\hbar}, \\ \theta = 2k_z/\sqrt{r}, \quad (10)$$

are the components of the dimensionless wave-vector and the Josephson's phase. For the dimensionless noise $\bar{\zeta}_p^*(u) \equiv \tau_0 \zeta_p(t)$ the correlator looks like

$$\langle \bar{\zeta}_p^*(u_1) \bar{\zeta}_p(u_2) \rangle = n_T \delta(u_1 - u_2), \quad n_T = \frac{T}{a_0}. \quad (11)$$

For the sake of simplicity let us first consider the one-dimensional case of hopping of fluctuation current through a chain of Josephson junctions. Then the TDGL equation in terms of dimensionless variables reads

$$d_u \psi_q(u) = -\nu(u) \psi_q(u) + \bar{\zeta}_q(u), \quad (12)$$

where $q = P_z \xi_c(0)/\hbar$ is the conserving dimensionless canonical momentum in z direction, $\bar{A}(u) = -e^* A(u) \xi_c(0)/\hbar$ is the dimensionless potential momentum corresponding to the dimensionless electric field

$$f(u) = e^* E(t) \tau_0 \xi_c(0)/\hbar = d_u \bar{A}(u). \quad (13)$$

The dimensionless decay rate is given by

$$2\nu(u) \equiv \omega(q + \bar{A}(u)) + \epsilon \quad (14)$$

and the kinetic dimensionless momentum for the one-dimensional task we are solving is

$$k_z = q + \bar{A}(u). \quad (15)$$

III. DERIVATION AND SOLUTION OF THE BOLTZMANN EQUATION

The TDGL equation Eq. (12) is a linear ordinary differential equation and one can easily check that its solution has the form

$$\begin{aligned} \psi_q(u) = & \left\{ \int_{u_2=0}^u \bar{\zeta}_q(u_2) \exp \left[\int_{u_1=0}^{u_2} \nu(u_1) du_1 \right] u_2 \right. \\ & \left. + \psi_q(0) \right\} \exp \left[- \int_{u_3=0}^u \nu(u_3) du_3 \right]. \end{aligned} \quad (16)$$

The averaging of the superconductor's order parameter over the noise gives the distribution of the mean number of particles with respect to the canonical momentum

$$\begin{aligned} n_q(u) = & \langle \psi_q^*(u) \psi_q(u) \rangle \\ = & n_T \int_0^u du_2 \exp \left[-2 \int_{u_2}^u du_1 \nu(u_1) \right] \\ & + n_q(u=0) \exp \left[-2 \int_0^u \nu(u_3) du_3 \right]. \end{aligned} \quad (17)$$

In case of constant electric field the vector-potential and the decay rate take the form

$$\bar{A}(u) = fu, \quad 2\nu(u) = \frac{r}{2} [1 - \cos(\phi + 2\mathcal{P}u)] + \epsilon, \quad (18)$$

where we have introduced the notations

$$\mathcal{P} \equiv \frac{f}{\sqrt{r}} = \frac{eE_z \tau_0 s}{\hbar}, \quad \phi \equiv \frac{P_z s}{\hbar} = \frac{2}{\sqrt{r}} q = \text{const.} \quad (19)$$

An elementary integration of the upper equation (17) in the limit $u \rightarrow \infty$, using the trigonometric relation

$$\sin(\phi + 2\mathcal{P}\tilde{u}) - \sin \phi = 2 \cos(\phi + \mathcal{P}\tilde{u}) \sin(\mathcal{P}\tilde{u})$$

gives

$$\begin{aligned} \frac{n(\theta)}{n_T} = & \int_0^\infty d\tilde{u} \\ & \times \exp \left[- \left(\epsilon + \frac{r}{2} \right) \tilde{u} + \frac{r}{2\mathcal{P}} \sin(\mathcal{P}\tilde{u}) \cos(\theta - \mathcal{P}\tilde{u}) \right]. \end{aligned} \quad (20)$$

Here we have turned to a new integration variable $\tilde{u} = u - u_2$, see Eq. (17), which describes the birth of fluctuation Cooper pairs in the moment $\tau_0 \tilde{u}$ before the time of observation $t = \tau_0 u$. In such a case the integrant gives the age distribution of fluctuation Cooper pairs. The periodical dependence $\sin(\theta + 2\mathcal{P}\tilde{u})$ on time \tilde{u} is a reflection of the Bloch oscillations of the free charged particles which move in a periodical potential and constant electric field.

In case of weak electric fields $\sin(\mathcal{P}\tilde{u}) \approx \mathcal{P}\tilde{u}$ we obtain the equilibrium distribution density function

$$\bar{n}(\theta) = \frac{n_T}{\frac{r}{2} (1 - \cos \theta) + \epsilon}. \quad (21)$$

The reconstruction of the plane components of the momentum leads to the general formula of Rayleigh-Jeans for the equilibrium distribution of the particles

$$\bar{n}_p = \frac{T}{\frac{p_x^2 + p_y^2}{2m_{ab}} + \frac{\hbar^2}{m_c^2 s^2} (1 - \cos \frac{p_z s}{\hbar}) + a_0 \epsilon} = \frac{T}{\varepsilon(\mathbf{p}) - \mu}. \quad (22)$$

Hereby formally we may identify the chemical potential with the Landau parameter $\mu = -a_0 \epsilon$. In this sense the critical temperature T_c sets the point of annulation of the chemical potential and fixes the beginning of Bose-Einstein condensation of the fluctuation Cooper pairs. It may be easily checked that the momentum distribution of the particles obeys the master equation of Boltzmann

$$d_t n_{\mathbf{P}}(t) = - \frac{n_{\mathbf{P}}(t) - \bar{n}_{\mathbf{P}-e^* \mathbf{A}}(t)}{\tau_{\mathbf{P}-e^* \mathbf{A}}} = - \frac{n_{\mathbf{P}}(t)}{\tau_{\mathbf{P}-e^* \mathbf{A}}} + \frac{n_T}{\tau_0}, \quad (23)$$

where the decay rate is proportional to the kinetic energy measured from the chemical potential

$$\frac{1}{\tau_{\mathbf{P}-e^* \mathbf{A}}} = \frac{2}{\tau_0} \nu(\mathbf{P} - e^* \mathbf{A}(t)) = \frac{\varepsilon(\mathbf{p}) + a_0 \epsilon}{a_0 \tau_0}. \quad (24)$$

In such a way we derived the Boltzmann equation for fluctuation Coper pairs for out-of-plane transport of a layered superconductor. The argument of the distribution and decay rate is the time-dependent kinetic momentum. The direct derivation of Boltzmann equation directly from the TDGL theory is too long and contains a lot of technical details.⁶

In case of zero electric field the relaxation rate of the order parameter with zero value of the wave vector runs through a typical slowing down

$$\tau(\epsilon) = \tau_{p=0} = \frac{\tau_0}{\epsilon}. \quad (25)$$

This is the characteristic time of “drying” at $\epsilon > 0$ of the spatially homogeneous Bose condensate with an order parameter $\psi(\mathbf{r}) = \text{const}$. If we consider the distribution depending on the kinetic momentum we obtain from TDGL theory the standard form of the Boltzmann equation derived in 1872

$$\begin{aligned} \partial_t n(\mathbf{p}, t) + e^* \mathbf{E}(t) \cdot \partial_{\mathbf{p}} n(\mathbf{p}, t) \\ = -\frac{n(\mathbf{p}, t) - \bar{n}(\mathbf{p})}{\tau(\mathbf{p})} = -\frac{n(\mathbf{p}, t)}{\tau(\mathbf{p})} + \frac{n_T}{\tau_0}. \end{aligned} \quad (26)$$

In our dimensionless variables for the static case this equation reads

$$2\mathcal{P}d_\theta n(\theta) = -\left[\frac{r}{2}(1 - \cos\theta) + \epsilon\right] n(\theta) + n_T, \quad (27)$$

which solution is Eq. (20).

In such a way we derived the Cooper pair Stoss-integral in the framework of the TDGL theory, the result is formally an energy dependent τ -approximation with a constant birth rate in momentum space

$$\frac{\bar{n}(\mathbf{p})}{\tau(\mathbf{p})} = \frac{n_T}{\tau_0}. \quad (28)$$

This integral describes collisions between normal particles and creation of Cooper pairs and back process of the decay of the condensate particles. We wish to point out that the TDGL equation is a diffusion-like equation which does not lead to quasi-classical dynamics of quasi-particles.⁷ This shows that Boltzmann equation has broader applicability than arguments used in his textbook presentations.

Let us now turn back to the analysis of the distribution function. In the continual limit when the coherence length significantly exceeds the lattice constant $\xi_c(0) \gg s$ or in other words when the LD parameter is big enough $r \gg 1$ we have to consider small Josephson's phase $|\theta| \ll 1$ and weak fields $|\mathcal{P}\tilde{u}| \ll 1$. The distribution over the kinetic momentum Eq. (20) then becomes^{7,8}

$$n(k_z) = n_T \int_0^\infty \exp\left\{-\left(\epsilon + k_z^2\right)\tilde{u} + k_z f \tilde{u}^2 - \frac{1}{3} f^2 \tilde{u}^3\right\} d\tilde{u}. \quad (29)$$

While in Refs. 7,8 are analyzed bulk superconductors in the present work we consider a layered superconductor with Josephson coupling between the layers for which we apply LD model. The last formula may be derived directly from the Boltzmann equation applied in case of parabolic dispersion $\varepsilon(p_z) \approx p_z^2/2m_c$. After we have investigated the momentum distribution here in the next section we will obtain a general formula for the density of the electrical current.

IV. FLUCTUATION CURRENT

Let us start our analysis with the one-dimensional case as the results may be easily extrapolated for the case of

higher dimensions. The density of the current

$$j_z = \sum_{p_z} e^* v(p_z) \frac{n(p_z)}{L} \quad (30)$$

is a product of the particles' charge e^* , their density $n(p_z)/L$ and velocity

$$v(p_z) = \frac{\partial \varepsilon}{\partial p_z} = \frac{\hbar}{m_c s} \sin \theta. \quad (31)$$

This formula is a special case of the general procedure for derivation of the current density in the framework of the GL's theory (Ref. [9], sec. 115; Ref. [10], sec. 45)

$$j(\mathbf{r}) = -\frac{\delta}{\delta A(r)} \int \psi^*(\mathbf{r}) \varepsilon(-i\hbar \nabla - e^* A(\mathbf{r})) \psi(\mathbf{r}) d^3x, \quad (32)$$

where after the functional integration the vector potential is put to be spatially homogeneous and the order parameter takes the form of a plane wave

$$\psi(\mathbf{r}) = \sqrt{\frac{n_p}{V}} e^{i\mathbf{p} \cdot \mathbf{r}/\hbar}, \quad n_p = \int |\psi(\mathbf{r})|^2 d^3x, \quad V = \int d^3x. \quad (33)$$

In the one-dimensional case, imagine current along a chain of Josephson junctions, if we use the relation

$$\sum_{p_z} = \frac{L}{2\pi\hbar} \int dp_z = L \int_{-\pi}^{\pi} \frac{d\theta}{2\pi s}, \quad (34)$$

the substitution of the Boltzmann equation's solution Eq. (20) in the general formula for the current Eq. (30) gives

$$\begin{aligned} j_z^{(1D)} &= \frac{e^* r T}{4\pi\hbar} \int_{-\pi}^{\pi} \sin \theta d\theta \int_0^\infty \exp\left\{-\left(\epsilon + \frac{r}{2}\right)\tilde{u}\right. \\ &\quad \left.+ \frac{r}{2\mathcal{P}} \sin(\mathcal{P}\tilde{u}) \cos(\theta - \mathcal{P}\tilde{u})\right\} d\tilde{u}. \end{aligned} \quad (35)$$

Here we have taken into account that $\theta = p_z s/\hbar$ and $a_0 n_T = T$, see Eq. (5) and Eq. (11). The averaging with respect to the Josephson's phase θ can be expressed by the modified Bessel functions¹¹

$$I_m(z) = (-1)^m \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \cos m\theta e^{-z \cos \theta} = i^{-m} J_m(iz). \quad (36)$$

Thus the formula for the current reads

$$j_z^{(1D)} = \frac{e^* r T}{2\hbar} \int_0^\infty d\tilde{u} e^{-(\epsilon+r/2)\tilde{u}} \sin(\mathcal{P}\tilde{u}) I_1\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (37)$$

In order to calculate the out-of-plane component of the current j_z in the LD model we have to consider the complementary decay rate excited by the kinetic energy in the xy plane, i.e. ab CuO₂ plane. For this reason formula Eq. (14) has to be modified as follows

$$2\nu = \frac{\varepsilon(\mathbf{p}) + a_0(\epsilon)}{a_0} = \frac{r}{2}(1 - \cos \theta) + \eta + \epsilon, \quad (38)$$

where according to Eq. (10)

$$\eta \equiv \frac{p_x^2 + p_y^2}{2m_{ab}a_0} = k_x^2 + k_y^2 \quad (39)$$

is the dimensionless in-plane kinetic energy limited by a cut-off parameter c ; for more details see, for example, Ref. 2. We have to sum over the kinetic momentums in the ab -plane

$$\int \int \frac{dp_x dp_y}{(2\pi\hbar)^2} f\left(\frac{p_x^2 + p_y^2}{2m_{ab}a_0}\right) = \frac{1}{4\pi\xi_{ab}^2(0)} \int_0^c d\eta f(\eta). \quad (40)$$

This additional integration with respect to the two-dimensional degrees of freedom taken into account in Eq. (37) gives the replacement

$$e^{-\epsilon\tilde{u}} \rightarrow \frac{e^{-\epsilon\tilde{u}}}{4\pi\xi_{ab}^2(0)} \int_0^c e^{-\eta\tilde{u}} d\eta = \frac{e^{-\epsilon\tilde{u}}(1 - e^{c\tilde{u}})}{4\pi\xi_{ab}^2(0)\tilde{u}}. \quad (41)$$

Then the expression for the fluctuation current takes the final form

$$j_z(\bar{\epsilon}, \mathcal{P}) = \frac{erT}{4\pi\hbar\xi_{ab}^2(0)} \int_{\tilde{u}=0}^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} \frac{1 - e^{-c\tilde{u}}}{\tilde{u}} \times \sin(\mathcal{P}\tilde{u}) I_1\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right), \quad (42)$$

where $\bar{\epsilon}$ means the self-consistent treatment of ϵ parameter, which will be analyzed in the next section.

V. MAXWELL-HARTREE SELF-CONSISTENT APPROXIMATION

Up to this moment we have neglected the influence of the nonlinear term in the GL theory. The coefficient standing in front of this term participates, for instance, in the equation for the equilibrium value of the order parameter above T_c

$$[a_0\epsilon + b|\psi|^2]\psi = 0. \quad (43)$$

The idea of the self-consistent approximation (SCA) is to replace the density of particles with its value averaged over all fluctuation modes. In such a way we obtain a self-consistent equation for the renormalized reduced temperature, see Ref. 2, Sec. 3.3

$$\bar{\epsilon} = \ln \frac{T}{T_c} + \frac{b}{a_0} n^{(3D)}(\bar{\epsilon}). \quad (44)$$

The idea is coming from the Maxwell treatment of the ring of Saturn – the first work on collective phenomena in physics. Maxwell concluded in 1856 that the ring cannot be a rigid object, but consists of “indefinite number of unconnected particles” and each of them is moving in the averaged gravitational field of the others. Analogously the fluctuation density of Cooper pairs renormalizes its chemical potential as shown in Eq. (44).

At first from Eq. (20) we calculate the one dimensional density of the Cooper pairs

$$n^{(1D)} = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi s} n(\theta). \quad (45)$$

Analogously to the calculation of the current Eqs. (30) and (34) now we have

$$n^{(1D)}(\bar{\epsilon}, \mathcal{P}) = \frac{n_T}{s} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} I_0\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (46)$$

The volume density can be obtained via summation of this one-dimensional density with respect to the in-plane degrees of freedom

$$n^{(3D)}(\bar{\epsilon}) = \int \int \frac{dp_x dp_y}{(2\pi\hbar)^2} n^{(1D)}\left(\bar{\epsilon} + \frac{p_x^2 + p_y^2}{2m_{ab}^*a_0}\right), \quad (47)$$

cf. the analogous transition between 2D and 3D case for in-plane conductivity of layered superconductors, Ref. 2, Sec. 2.3. The kinetic energy cut-off, Ref. 2, Eq. (11)

$$\frac{p_x^2 + p_y^2}{2m_{ab}^*} < a_0 c \quad (48)$$

leads to the same replacement as in Eq. (41) and for the 3D density we get

$$n^{(3D)}(\bar{\epsilon}, \mathcal{P}) = \frac{n_T}{4\pi s \xi_{ab}^2(0)} \int_{\tilde{u}=0}^{\infty} d\tilde{u} \frac{1 - e^{-c\tilde{u}}}{\tilde{u}} \times e^{-(\bar{\epsilon}+r/2)\tilde{u}} I_0\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (49)$$

Additionally we may represent the nonlinear parameter b in terms of the Ginzburg-Landau parameter $\kappa_{GL} = \lambda_{ab}(0)/\xi_{ab}(0)$ and the flux quantum Φ_0 , see Ref. 2, Eq. (165)

$$b = 2\mu_0 \left(\frac{\pi\hbar\kappa_{GL}}{\Phi_0 m_{ab}^*}\right)^2. \quad (50)$$

We also introduce the dimensionless Ginzburg number for a layered superconductor (cf. Ref. 2, Eq. (168))

$$\epsilon_{Gi}(T) = 2\pi\mu_0 \frac{T}{s} \left(\frac{\lambda_{ab}(0)}{\Phi_0}\right)^2 = \frac{2\mu_0\kappa_{GL}^2 e^2 \xi_{ab}^2(0) T}{\pi\hbar^2 s}. \quad (51)$$

In terms of the so introduced notations the self-consistent equation for the renormalized temperature reads

$$\bar{\epsilon} = \epsilon + \epsilon_{Gi} \int_0^{\infty} d\tilde{u} \frac{1 - e^{-c\tilde{u}}}{\tilde{u}} e^{-(\bar{\epsilon}+r/2)\tilde{u}} I_0\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right), \quad (52)$$

in agreement with the results of Puica and Lang Ref.5, Eqs. (11) and (17), where the temperature-dependent Ginzburg number $\epsilon_{Gi}(T)$ is denoted by gT . The solution $\bar{\epsilon}$ of equation (52) has to be substituted as an argument

in the formula for the current Eq. (42). This agreement demonstrates that it is possible to derive in the beginning an 1-dimensional (1D) formula for the Josephson chain and later on to generalize this approach to a 3-dimensional (3D) case of layered superconductor. In the next section we will analyze the modification of this result in case of out-of-plane external magnetic field.

VI. INFLUENCE OF A PERPENDICULAR MAGNETIC FIELD. MAGNETOCONDUCTIVITY

A. General case

Let us consider an external magnetic field also applied perpendicularly to the CuO_2 planes. In this case the variables can be separated and the task for the calculation of the current reduces to the 1D case Eq. (37), which we have already considered. For the two-dimensional movement the magnetic field arouses equidistant oscillator spectrum

$$\frac{\mathbf{p}_{ab}}{2m_{ab}^*} \rightarrow \hbar\omega_c \left(n + \frac{1}{2} \right), \quad \omega_c = \frac{e^* B_z}{m_{ab}^*}. \quad (53)$$

The degeneration rate is determined by the Landau subzone capacity B/Φ_0 , where $\Phi_0 = 2\pi\hbar/e^*$ is the magnetic flux quantum. For parametrization of the GL theory it is convenient to introduce the linear extrapolated upper critical field, represented via the coherence length in the ab -plane

$$B_{c2}(0) \equiv -T_c \left. \frac{B_{c2}(T)}{dT} \right|_{T_c} = \frac{\Phi_0}{2\pi\xi_{ab}^2(0)}. \quad (54)$$

For facilitation we will work with the dimensionless magnetic field

$$h \equiv \frac{B_z}{B_{c2}(0)} = \frac{\hbar\omega_c}{2a_0}, \quad (55)$$

with the help of which the dimensionless spectrum quantizes as

$$\eta \rightarrow (2n+1)h, \quad n = 0, 1, 2, 3, \dots \quad (56)$$

The maximal kinetic energy is reached at some big number N_c , which cuts off the summation over the Landau levels

$$\hbar\omega_c N_c = a_0 c, \quad 2hN_c = c. \quad (57)$$

The influence of the magnetic field reduces to the replacement of the integration over the kinetic energy to a restricted summation over the Landau levels

$$\int_0^c d\eta f(\eta) \rightarrow \Delta\eta \sum_{n=0}^{N_c-1} f((2n+1)h). \quad (58)$$

In particular, in the exponential dependence, which we have in Eq. (41), we have to sum up the limited geometric progression

$$\int_0^c d\eta e^{-\eta\tilde{u}} \rightarrow 2h \sum_{n=0}^{N_c-1} e^{-(2n+1)h\tilde{u}}, \quad (59)$$

which more easily can be represented as a subtraction of two infinite ones

$$\sum_{n=0}^{N_c-1} = \sum_{n=0}^{\infty} - \sum_{n=N_c}^{\infty}. \quad (60)$$

In this way the summation over the in-plane degrees of freedom simply reduces to the shift

$$\int_0^c d\eta e^{-\eta\tilde{u}} \rightarrow \frac{h}{\sinh h\tilde{u}} (1 - e^{-c\tilde{u}}). \quad (61)$$

Then the substitution described in Eq. (41) in the presence of external magnetic field takes the form

$$e^{-\epsilon\tilde{u}} \rightarrow \frac{e^{-\epsilon\tilde{u}}}{4\pi\xi_{ab}^2(0)} \int_0^c e^{-\eta\tilde{u}} d\eta \rightarrow \frac{e^{-\epsilon\tilde{u}}(1 - e^{-c\tilde{u}})h}{4\pi\xi_{ab}^2(0) \sinh h\tilde{u}}. \quad (62)$$

This is the recipe with which one may switch from the already solved one-dimensional task Eq. (37) to the three-dimensional. Comparison with the case of zero magnetic field shows that we may use the 3D formulas Eqs. (52) and (42), where we only have to make the substitution

$$\frac{1}{\tilde{u}} \rightarrow \frac{h}{\sinh(h\tilde{u})}. \quad (63)$$

In this way, in agreement with Ref. 5, Eqs. (13) and (14), we derive the final formulas for the self-consistent reduced temperature

$$\bar{\epsilon} = \epsilon + \epsilon_{G1} \int_0^\infty d\tilde{u} \frac{(1 - e^{-c\tilde{u}})h}{\sinh h\tilde{u}} e^{-(\bar{\epsilon}+r/2)\tilde{u}} I_0 \left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}} \right) \quad (64)$$

and the perpendicular to the CuO_2 planes density of the current

$$j_z(\bar{\epsilon}, \mathcal{P}, h) = \frac{e^2 \xi_c^2(0) E_z}{16\hbar s \xi_{ab}^2(0)} \int_{\tilde{u}=0}^\infty d\tilde{u} \frac{(1 - e^{-c\tilde{u}})h}{\sinh h\tilde{u}} e^{-(\bar{\epsilon}+r/2)\tilde{u}} \times \frac{\sin(\mathcal{P}\tilde{u})}{\mathcal{P}} I_1 \left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}} \right), \quad (65)$$

where we have used the relation

$$\frac{erT\mathcal{P}}{4\pi\hbar\xi_{ab}(0)} = \frac{e^2 \xi_c^2(0) E_z}{16\hbar s \xi_{ab}^2(0)}. \quad (66)$$

This explicit formula is one of the new results of the present work. Let us now analyze the magnetoconductivity in some characteristic particular examples.

B. Weak magnetic fields

Let us consider the application of the upper formulas in case of weak magnetic field $h \ll |\bar{\epsilon}|$. If we use the well-known summation of Euler-MacLaurin (cf. with Ref. 2, Eq. (23)) we have

$$\frac{x}{\sinh x} = 1 - \frac{1}{6}x^2 + \frac{7}{360}x^4 - \frac{31}{15120}x^6 + \dots \quad (67)$$

For weak magnetic fields we may use the approximation

$$\frac{h}{\sinh h\tilde{u}} \approx \frac{1}{\tilde{u}} - \frac{1}{6}h^2\tilde{u}. \quad (68)$$

The substitution of this decay into Eq. (65) leads to the special expression for the current in the presence of weak magnetic fields

$$j_z(\bar{\epsilon}, \mathcal{P}, h) = j_z(\bar{\epsilon}, \mathcal{P}) - \frac{erTh^2}{24\pi\hbar\xi_{ab}^2(0)} \int_{\tilde{u}=0}^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} (1 - e^{-c\tilde{u}})\tilde{u} \times \sin(\mathcal{P}\tilde{u}) I_1\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (69)$$

The coefficient in front of the integral over the reducing term may be expressed via the coherent lengths and the lattice constant as follows

$$\frac{erTh^2}{24\pi\hbar\xi_{ab}^2(0)} = \frac{2\pi eT\xi_c^2(0)}{3\hbar s^2} \left(\frac{\xi_{ab}^2(0)B_z}{\Phi_0}\right)^2. \quad (70)$$

The experimental data fitting with this formula could lead to more precise specification of $\xi_c(0)$ for the investigated sample. The procedure Eq. (68) applied to Eq. (64) ends in a self-consistent equation for the reduced temperature in case of weak magnetic fields

$$\bar{\epsilon}(h) \approx \epsilon - \epsilon_{Gi} \frac{h^2}{6} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} (1 - e^{-c\tilde{u}}) \times \tilde{u} I_0\left(\frac{r \sin \mathcal{P}\tilde{u}}{2\mathcal{P}}\right). \quad (71)$$

In the next subsection we will analyze the case of values of the magnetic field close to the phase curve $B_{c2}(T)$, where the influence of the fluctuation pairs is most vividly expressed.

C. Strong fields close to $B_{c2}(T)$

Close to the phase curve it is more convenient to account the reduced temperature in regards of the phase transition temperature, which is a function of the external magnetic field $T_{c2}(B)$. Comparison with Eq. (54) reads

$$\epsilon_h = \frac{T - T_{c2}(B)}{T_c} = \epsilon + h, \quad |\epsilon_h| \ll h. \quad (72)$$

In the case of strong magnetic fields it is more appropriate to use the 1D formula Eq. (37), where we have to add a summation over the Landau levels, taking into account the degeneration and the cut off

$$j_z^{(3D)}(\bar{\epsilon}, \mathcal{P}, h) \approx \frac{B}{\Phi_0} \sum_{n=0}^{\infty} j_z^{(1D)}(\bar{\epsilon}_h + 2nh, \mathcal{P}). \quad (73)$$

In close vicinity of the upper critical field $|\bar{\epsilon}| \ll h$ in the summation the share of the lowest Landau level dominates over the others and its contribution gives

$$j_z^{(3D)} \approx \frac{e^* r T B}{2\hbar \Phi_0} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}_h+r/2)\tilde{u}} \sin(\mathcal{P}\tilde{u}) I_1\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (74)$$

Similar procedure applied to the number of particles' density Eq. (46), see also Eq. (52),

$$\bar{\epsilon}_h = \epsilon_h + \frac{bB}{a_0\Phi_0} n^{(1D)}(\bar{\epsilon}_h, \mathcal{P}) \quad (75)$$

determines the equation for the self-consistent reduced temperature

$$\bar{\epsilon}_h = \epsilon_h + 2h\epsilon_{Gi} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}_h+r/2)\tilde{u}} I_0\left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}}\right). \quad (76)$$

Close to the upper critical magnetic field the fluctuations are practically one-dimensional and such a reduction of the space dimensionality significantly amplifies the fluctuation phenomena.

Next we are going to derive an explicit formula for the differential conductivity in the most general case of LD model and analyze it in the particular cases of both weak and strong magnetic fields. The physical conditions of possible occurrence of negative differential conductivity will be discussed.

VII. DIFFERENTIAL CONDUCTIVITY. POSSIBLE NDC

The non-linear effects over the conductivity $\sigma_{zz}(E_z) \equiv j_z/E_z$ could be best understood in the investigation of the differential conductivity

$$\sigma_{\text{diff}}(E_z) \equiv \frac{dj_z}{dE_z}. \quad (77)$$

In order to derive it we have to differentiate the general formula for the current in an external magnetic field Eq. (65) and use the well-known properties of the modified Bessel functions

$$2 \frac{dI_m(z)}{dz} = I_{m+1}(z) + I_{m-1}(z) \quad (78)$$

which can be found in any special functions manual, for example see Ref. 11 or Ref. 12 and references therein. We turn to differentiation with respect to the dimensionless

electric field \mathcal{P} , taking into account the value of the decay rate τ_0 that follows from the BCS theory

$$\frac{\partial \mathcal{P}}{\partial E_z} = \frac{e\sigma\tau_0}{\hbar}, \quad \tau_0^{(\text{BCS})} = \frac{\pi\hbar}{16T_c}. \quad (79)$$

From Eq. (65) it is straight forward that the electric field-dependent part in the differential conductivity is given by the expression

$$\begin{aligned} & \frac{\partial}{\partial \mathcal{P}} \left[\sin(\mathcal{P}\tilde{u}) I_1 \left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}} \right) \right] \\ &= \frac{r\tilde{s}}{4\mathcal{P}^2} (\mathcal{P}\tilde{u}\tilde{c} - \tilde{s}) (I_0 + I_2) + \tilde{u}\tilde{c}I_1, \end{aligned} \quad (80)$$

where for the sake of brevity we have introduced the notations

$$\tilde{s} = \sin(\mathcal{P}\tilde{u}), \quad \tilde{c} = \cos(\mathcal{P}\tilde{u}), \quad I_m = I_m \left(\frac{r \sin(\mathcal{P}\tilde{u})}{2\mathcal{P}} \right). \quad (81)$$

Having in mind the considerations above it can be easily shown that the general formula for the differential conductivity takes the final form

$$\begin{aligned} \sigma_{\text{diff}}(\bar{\epsilon}, \mathcal{P}, h) &= \frac{e^2 \xi_c^2(0)}{16\hbar s \xi_{ab}^2(0)} \int_{\tilde{u}=0}^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} \frac{(1 - e^{-c\tilde{u}})h}{\sinh h\tilde{u}} \\ &\times \left[\frac{r\tilde{s}}{4\mathcal{P}^2} (\mathcal{P}\tilde{u}\tilde{c} - \tilde{s}) (I_0 + I_2) + \tilde{u}\tilde{c}I_1 \right]. \end{aligned} \quad (82)$$

This is the equation we are going to use in order to analyze the presence of negative differential conductivity for given samples. Hereof we may derive the expressions for σ_{diff} in the special cases of weak and strong magnetic fields.

In the absence of external magnetic field the limit $h \rightarrow 0$ gives $h/\sinh h \rightarrow 1$ and we obtain

$$\begin{aligned} \sigma_{\text{diff}}(\bar{\epsilon}, \mathcal{P}) &= \frac{e^2 \xi_c^2(0)}{16\hbar s \xi_{ab}^2(0)} \int_{\tilde{u}=0}^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} \frac{1 - e^{-c\tilde{u}}}{\tilde{u}} \\ &\times \left[\frac{r\tilde{s}}{4\mathcal{P}^2} (\mathcal{P}\tilde{u}\tilde{c} - \tilde{s}) (I_0 + I_2) + \tilde{u}\tilde{c}I_1 \right], \end{aligned} \quad (83)$$

which may be derived straight from Eq. (42).

For strong magnetic fields we take into account only the contribution of the lowest Landau level and the differentiation of Eq. (74) reads

$$\begin{aligned} \sigma_{\text{diff}}(\bar{\epsilon}_h, \mathcal{P}) &= \frac{\pi e^2 \xi_c^2(0) B_z}{4\hbar s \Phi_0} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}_h+r/2)\tilde{u}} \\ &\times \left[\frac{r\tilde{s}}{4\mathcal{P}^2} (\mathcal{P}\tilde{u}\tilde{c} - \tilde{s}) (I_0 + I_2) + \tilde{u}\tilde{c}I_1 \right]. \end{aligned} \quad (84)$$

If we want to return to the one-dimensional differential conductivity for strong magnetic fields close to the phase curve we only have to omit the number of magnetic threads B_z/Φ_0 in the upper equation, which gives

$$\begin{aligned} \sigma_{\text{diff}}^{(1D)}(\bar{\epsilon}, \mathcal{P}) &= \frac{\pi e^2 \xi_c^2(0)}{4\hbar s} \int_0^{\infty} d\tilde{u} e^{-(\bar{\epsilon}+r/2)\tilde{u}} \\ &\times \left[\frac{r\tilde{s}}{4\mathcal{P}^2} (\mathcal{P}\tilde{u}\tilde{c} - \tilde{s}) (I_0 + I_2) + \tilde{u}\tilde{c}I_1 \right]. \end{aligned} \quad (85)$$

Of course the calculated derivative $(\partial j_z / \partial E_z)_{\bar{\epsilon}}$ does not describe the whole effect. We have to take into account also $(\partial \bar{\epsilon} / \partial E_z)_{\bar{\epsilon}}$ and the increase of the sample's temperature above the ambient temperature

$$\mathcal{R}_{\text{therm}}(T - T_{\text{amb}}) = \mathcal{V} j_z E_z, \quad (86)$$

due to the thermal resistance. The “chemical” reaction of pairing of normal charge carriers $e + e \rightarrow e^*$ creates also a decreasing of the number of normal charge carriers, and in the Drude formula, for example, we have to take into account¹ the density of state's corrections (DOS) $\sigma_{\text{norm}} = (n_e - 2n(\bar{\epsilon}, \mathcal{P}))e^2\tau_{\text{norm}}/m_e$.

All those effects are however smaller compared to the dominating Aslamazov-Larkin conductivity which qualitatively can describe the appearance of new physical effects such as NDC, for example.

In order to better understand the nature of NDC in supercooling regime in the next section we will analyze the Green functions of the Boltzmann equation.

VIII. GREEN FUNCTIONS OF THE BOLTZMANN EQUATION

Now let us paint some Cooper pairs in *green* in order to trace their motion i.e. in order to analyze the influence of the random noise on the momentum distribution of the Cooper pairs we will replace the constant income term in the Stoss-integral n_0/τ_0 with a δ -function inhomogeneous term in Eqs. (23) and (26)

$$\frac{n_0}{\tau_0} \rightarrow \frac{2\pi\hbar}{L} \delta(t) \delta(p - p_0). \quad (87)$$

For simplicity we will consider 1D case.

Before the moment of creation there are no Cooper pairs $n_p(t < 0) = 0$. Then immediately after the influence we have δ -like initial distribution

$$n_p(t = +0) = \delta_{p,p_0} = \frac{2\pi\hbar}{L} \delta(p - p_0). \quad (88)$$

This distribution is normalized to have one Cooper pair in the initial moment

$$\mathcal{N}(t) = \sum_p n_p(t) = L \int \frac{dp}{2\pi\hbar} n_p(t), \quad \mathcal{N}(+0) = 1 \quad (89)$$

born by thermal fluctuations; pictorially speaking “by the sea foam.” According to the separation of variables in the Boltzmann equation Eq. (23) in every momentum point the evolution of the distribution function is independent

$$n_p(u = t/\tau_0) = n_p(0) \exp \left\{ - \int_0^u 2\nu(u_3) du_3 \right\}. \quad (90)$$

The substitution here of the decay rate from Eq. (18) gives

$$\frac{\mathcal{N}(u)}{\mathcal{N}(0)} = \exp \left\{ - \left(\epsilon + \frac{r}{2} \right) u + \frac{r \sin(\mathcal{P}u)}{2\mathcal{P}} \cos(\theta - \mathcal{P}u) \right\}. \quad (91)$$

It is instructive to consider the 3D limit case of $|\epsilon| \ll r$; formally one can consider the unrealistic for the cuprates case of $r \gg 1$. For small angles

$$\theta = \frac{2sk}{\sqrt{r}} \ll 1, \quad \mathcal{P} = \frac{f}{\sqrt{r}}, \quad \mathcal{P}u \ll 1 \quad (92)$$

the Taylor expansion of the trigonometric functions in Eq. (91) gives

$$\frac{\mathcal{N}(u)}{\mathcal{N}(0)} = \exp \left\{ -(\epsilon + q^2)u + qfu^2 - \frac{1}{3}f^2u^3 \right\}. \quad (93)$$

The simplest example is to consider zero initial momentum $q = \xi_c(0)p_0/\hbar = 0$. In the supercooling regime where $\epsilon < 0$ in the beginning the number of Cooper pairs increases. It is a typical lasing process – increasing of the number of coherent Bose particles in one mode; imagine a drop of rain increasing by the condensation in a humid atmosphere. Finally, however, the $\frac{1}{3}f^2u^3$ term dominates in the argument of the exponent and $\mathcal{N}(\infty) = 0$; one can say that the droplet is evaporated by the big kinetic energy like a meteorite falling in the earth atmosphere. The droplets are not smeared during their lifetime

$$n_p(t) = \frac{2\pi\hbar}{L} \frac{\mathcal{N}(u)}{\mathcal{N}(0)} \delta(p - p_0). \quad (94)$$

They have just a drift in the kinetic momentum space. This drift related to the electric current is analogous to the rain fall created by the earth acceleration.

$$n(\mathbf{p}, t) = \frac{2\pi\hbar}{V} \frac{\mathcal{N}(u)}{\mathcal{N}(0)} \delta(\mathbf{p} - \mathbf{p}_0 - e^*\mathbf{E}t). \quad (95)$$

In upper *Green* function solution of the Boltzmann equation Eq. (26) we have recovered the 3D variables.

Now we can understand qualitatively the mechanism of creation of NDC for small electric fields in a supercooled superconductor. Let us take for illustration $p_0 = 0$ case in Eq. (93). For $\epsilon < 0$, $h = 0$, and $|\epsilon| \ll r$ this function has a maximum⁷

$$\frac{\mathcal{N}(u_{\max})}{\mathcal{N}(0)} = \exp \left\{ \frac{2}{3} \frac{(-\epsilon)^{3/2}}{f} \right\} \quad (96)$$

at

$$u_{\max} = \frac{\sqrt{(-\epsilon)}}{f}. \quad (97)$$

The maximal increase of the “weight” of the Bose droplet $\mathcal{N}(u_{\max})/\mathcal{N}(0)$ can reach big values for small enough electric fields

$$f \ll (-\epsilon)^{3/2}, \quad (98)$$

or

$$e^*E\tau(\epsilon)\xi(\epsilon)/\hbar \ll 1, \quad \tau(\epsilon) = \frac{\tau_0}{|\epsilon|}, \quad \xi(\epsilon) = \frac{\xi(0)}{|\epsilon|^{1/2}}. \quad (99)$$

Decreasing of the electric field increases the maximal size of the Cooper pair droplets and the electric current which is proportional to the number of particles in the droplets. This precursor of the Bose condensation creates the NDC. Our preliminary numerical calculations for $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (Y:123) taking the parameters from Ref. 4: $s = 1.17$ nm, $\xi_{ab}(0) = 1.2$ nm, $\xi_c(0) = 0.14$ nm, $T_c = 92.01$ K, $T_c^{(\text{ren})} = 86.9$ K, $\kappa_{\text{GL}} = 70$, $1/\sigma_{zz}^{(\text{norm})} = 5$ m Ω cm, $c = 0.5$ demonstrated existence of NDC; this gives the hope that NDC can be experimentally observed for out-of-plane transport in Y:123.

IX. ANALYSIS, DISCUSSION AND CONCLUSION

Let us start with the formal discussion. In case of big enough overcooling $\epsilon + r/2 < 0$, when $\epsilon < 0$ and $|\epsilon| > r$ the integral for the current Eq. (42) is divergent. This means that the electric field cannot prevent the appearance of a superconducting condensation, as is the case of volume superconductors. The reason for this is that the one-dimensional zone in the energy spectrum Eq. (3) has a finite width a_0r , in contrast to the LD model, which shall be obtained if we take the continual limit of $r \gg 1$. However, if the LD parameter is small enough we have to take into account the interaction between the fluctuation modes, described by the nonlinear term in the GL theory. These effects become important when the reduced temperature ϵ is comparable to the Ginzburg number for layered superconductors

$$\epsilon_{\text{Gi}}(T_c) = \frac{1}{4\pi\xi_{ab}^2(0)s\Delta C}, \quad (100)$$

where (see Ref. 2, Sec. 3.3, Eq. 169) ΔC is the jump of the heat capacity, extrapolated from the critical behaviour of the heat capacity $C(T)$ as a fitting parameter, or can be evaluated by the electrodynamic properties

$$T_c\Delta C = \frac{1}{8\pi^2\mu_0} \left(\frac{\Phi_0}{\lambda_{ab}(0)\xi_{ab}(0)} \right)^2. \quad (101)$$

In the Maxwell-Hartree SCA the total volume density of the fluctuation Cooper pairs leads to an effective heating (see Ref. 2, Sec. 3.3, Eqs. (164) and (173)). As a whole, inclusion of the SCA does not change the qualitative behavior of the obtained results. In overcooled superconductors $\epsilon < 0$, for example, we expect the appearance of a negative differential conductivity (NDC) with the decreasing of the constant electric field. Whether this will lead to formation of a domain structure of normal and superconducting layers or to existence of electric oscillations is a question of further analysis. Our theory, however, predicts that this phase transition when $T < T_c$ will take place via annulation of the differential conductivity. In this respect the layered superconductors give an advantage as, because of the strong anisotropy, the normal conductivity perpendicular to the layers is too

small. Thus the sample will not heat intensively, which gives the opportunity to be observed the nonlinear effects of a strong electric field over the fluctuation conductivity. These nonlinear effects can be observed as an amplification of the current harmonics generation as we reach the critical region. Systematical investigation of these harmonics will lead to specification of the parameters in the TDGL theory and to clarification of the kinetics of the superconducting order parameter. Here we would like to consider the possible application of our theory for different layered cuprates. For instance, for the extremely anisotropic $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ (Bi:2212) the value of the LD parameter r is exceptionally small. The heating from the electric field applied in the “dielectric” z -direction is very weak, but the area where current harmonics can be observed is very narrow as well. Because of this their observation would require samples of extraordinary quantity. Wider and easier for observation would be the nonlinear region for the cuprates with a moderate anisotropy like Y:123. For most favorable we consider the cuprate $\text{Ta}_2\text{Sr}_2\text{CuO}_6$ (Ta:2201) where a coherent Fermi surface is observed. When the anisotropy is small and r parameter bigger, the half-width of the z -zone $\frac{1}{2}a_0r = \hbar^2/m_c^2s^2$ is bigger and then the electric field E_z causes more significant increase of the kinetic energy of the Cooper pairs $\frac{1}{2}a_0r(1 - \cos p_z s/\hbar)$. Because of this, for instance, the overcooling region when $\epsilon < 0$ and $\mathcal{P} \neq 0$ would be wider and more easily observed. Exactly, in such more easily reached by the experiment conditions, we hope to observe the annulation of the total differential conductivity. For this purpose submicron high qualitative films of Ta:2201 are necessary to be used. The investigation of the harmonics generation above T_c is a useful beginning for the systematical research of the nonlinear effects of the strong electric field over the fluctuation current in z -direction.

The experiment which we advocate is principally very simple, but according to the best of our knowledge not done yet. The fluctuation conductivity have to be investigated in a *voltage biased circuit*; a constant (DC) voltage has to be applied. A significantly smaller AC voltage has also to be applied to the sample. The DC current gives the conductivity and a measurement with a smaller AC current gives the differential conductivity. The circuit should be similar to the devices invented for investigation of the NDC in tunnel diodes. The DC voltage has to be added to the normal phase above T_c and the simplest experiment which we suggest is to measure the temperature dependence of the AC current (differential conductivity) as a function of the temperature. The theory reliably predicts annulation of the differential conductivity at cooling below T_c . What exactly will happen at further cooling is difficult to be predicted experimentally: it could be NDC if the space homogeneity is conserved, creation of layered domain structure of normal and superconducting layers or complicated dynamic phenomena. Different instabilities can be triggered by small perturbations including the nature of contacts and chemical treatment of the surfaces. One thing is sure: indispensably, new physics will

happen in the area below the annulation of the differential conductivity. The realization of the simplest possible scenario of creation of NDC will open the technological perspective to create a new type of high-frequency oscillators operating in the THz gap.¹³ While in Ref. 13 is analyzed the case of bulk superconductors in the present work we demonstrated that for electric currents applied in the “dielectric” c -direction the dissipated power could be orders of magnitude smaller and perhaps Josephson coupled superconductors can realize the first technical applications of NDC in superconductors; it was one of the motivations to perform the present work.

Few words we wish to add concerning the theoretical notions used in the present work. It seems very strange that Boltzmann kinetic approach has still limited usage in the physics of paraconductivity as it was for the normal conductivity hundred years ago. Just Green functions of the Boltzmann equation give the transparent picture what is happening in homogeneous electric fields. Separation of the variables in optical gauge with zero electric potential $\varphi = 0$ reduces the kinetic equation to an ordinary differential equation. Returning to the kinetic momentum space gives the drift of the fluctuation Cooper pairs created by the electric field.

Let us continue the discussion of physics in supercooling regime. Near to the annulation point of the differential conductivity the paraconductivity is not a small excess perturbation but will become comparable with the fluctuation conductivity. In the suggested experiment of applied DC electric field and small AC voltage one can observe in principle the AC component of the fluctuation magnetization M_z . This physical effect is described by magnetoelectric susceptibility $\chi_{\text{ME}} = \partial M_z / \partial E_z$; this theory will be the subject of a future research. Measurements of magnetization and paraconductivity in one and the same sample can lead to exact determination of the time constant τ_0 . This is the most important parameter of TDGL theory which reveals in part the nature of superconductivity and pairing mechanism. The $\pi/8$ multiplier in Eq. (6) is only weak coupling BCS result, but as we already mentioned, for references of relevant works see Sec 4.2 of the review,² the experimental accuracy now is not enough to observe reliably any deviations from this BCS $\pi/8$ value. There is no consensus that BCS theory is directly applicable for high- T_c cuprates and one might expect some correction multiplier τ_{rel} to the life time of the order of one. The relative life time can give important information for the pairing mechanisms in cuprates.

From qualitatively point the numerical value of τ_0 is not so important because only fixes the scale for the graphical presentation of the experimental data. In the chosen dimensionless units the current voltage curves should be universal and the most important property of the current voltage curves is the predicted annulation of the differential conductivity at supercooling below T_c . Demonstration of this annulation and possible NDC in out-of-plane transport can trigger significant technical applications¹³ and we hope that experimental search of

NDC in superconductors can start in the nearest future.

D. Damianov, V. Mishonova and W. Lang are highly appreciated.

Acknowledgments

The scientific discussions, support, critical reading of the manuscript, and correspondence with A. Varlamov,

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